Enrollment No: ______________________Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name: Mathematical Methods - I

Subject Code: 5SC03MAM1 **Branch: M.Sc. (Mathematics)**

Semester: 3 Date: 29/11/2018 **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the following questions** **(07)**

a. Write Dirichle's conditions for Fourier series. (02)

b. If $F(\lambda)$ is the Fourier transform of $f(x)$, then prove that the Fourier transform $f(x-a)$ is $e^{i\lambda a}F(\lambda)$. (02)

c. Check whether given function is even or odd. (01)

$$f(x)=\begin{cases}-\sin x, & -\pi\le x\le 0\\ \sin x, & 0\le x\le \pi\end{cases}.$$

d. If $F_c\big(f(ax)\big)=k\,F_c\left(\frac{\lambda}{a}\right)$, then $k=$ ______. (01)

e. Find value of the Fourier coefficient a_n for the $2-$ periodic function $f(x)=x$, $x\in(-1,1)$. (01)

Q-2 **Attempt all questions** **(14)**

a. Find Fourier series expansion of $f(x)=x, 0<x<2\pi$. (05)

b. Using Fourier sine integral, show that (05)

$$\int_0^\infty \frac{1-\cos\pi\lambda}{\lambda}\sin x\lambda\ d\lambda=\begin{cases}\frac{\pi}{2}, & when\ \ 0<x<\pi\\ 0, & when\ x>\pi\end{cases}.$$

c. Expand $f(x)=\begin{cases}\frac{1}{4}-x, if\ 0<x<\frac{1}{2}\\ x-\frac{3}{4}, if\ \frac{1}{2}<x<1\end{cases}$, as the Fourier series of sine terms. (04)

OR

Q-2 **Attempt all questions** **(14)**

a. Find Fourier series expansion of $f(x)=\begin{cases}-k, if-\pi<x<0\\ k, if\ \ 0<x<\pi\end{cases}$. (05)

b. Using Fourier cosine integral, prove that $\int_0^\infty\frac{\cos\lambda x}{\lambda^2+1}d\lambda=\frac{\pi}{2}e^{-x}, x\ge 0$. (05)

c. Express $f(x)=x$ as a half-range cosine series in $0<x<2$. (04)

Q-3 **Attempt all questions** **(14)**

a. State and prove convolution theorem of Fourier transform. (05)

	b.	Find the Fourier series expansion of $f(x) = 2x - x^2$ in $(0, 3)$.	(05)
	c.	Find the Fourier transform of $f(x) = \begin{cases} x, \lvert x\rvert < a \\ 0, \lvert x\rvert > a \end{cases}$.	(04)

OR

Q-3		**Attempt all questions**	**(14)**
	a.	Find the Fourier transform of $\frac{\partial^n u}{\partial x^n}$ of the function $u(x,t)$ assuming that u and its first $(n-1)$ derivatives with respect to x vanish as $x \to \pm\infty$.	(05)
	b.	If $f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ \sin x, & 0 \le x \le \pi \end{cases}$, then show that $f(x) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\cos 2nx}{4n^2-1}$.	(05)
	c.	Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, if\ \lvert x\rvert < 1 \\ 0 \quad\quad, if\ \lvert x\rvert > 1 \end{cases}$.	(04)

SECTION – II

Q-4		**Attempt the following questions**	**(07)**
	a.	Prove that $L[2^t] = \frac{1}{s - \log_e 2}$, if $s > \log_e 2$.	(02)
	b.	Evaluate $L^{-1}\left\{\frac{1}{\sqrt{s}} - \frac{7}{3s+2}\right\}$.	(02)
	c.	If $L^{-1}\{\bar{f}(s)\} = f(t)$, then $L^{-1}\left\{\frac{1}{s}\bar{f}(s)\right\} =$ _________.	(01)
	d.	State convolution theorem for Laplace transform.	(01)
	e.	Z-transform is linear. (True or False)	(01)
Q-5		**Attempt all questions**	**(14)**
	a.	Using Laplace transform, determine the solution of $\frac{d^2y}{dt^2} + 2\left(\frac{dy}{dt}\right) + 2y = 2$, $y(0) = 0, y'(0) = 1$.	(07)
	b.	If $f(t)$ is a periodic function with period T, then prove that $L[f(t)] = \frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)\ dt$.	(04)
	c.	Evaluate: $\int_0^{\infty} t\, e^{-3t} \cos t\ dt$.	(03)

OR

Q-5		**Attempt all questions**	**(14)**
	a.	Solve the following IBVP using the Laplace transform : PDE: $u_t = u_{xx}$, $0 < x < 1, t > 0$ BCs: $u(0,t) = 1, u(1,t) = 1$, $t > 0$ ICs: $u(x,0) = 1 + \sin \pi x$, $0 < x < 1$.	(07)
	b.	Find inverse Laplace transform of the $\bar{f}(s) = \frac{5s+3}{(s-1)(s^2+2s+5)}$.	(04)
	c.	Prove that if $L[f(t)] = \bar{f}(s)$, then $L[f^n(t)] = s^n\bar{f}(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - sf^{n-2}(0) - f^{n-1}(0)$.	(03)
Q-6		**Attempt all questions**	**(14)**
	a.	Find Z-transform of (1) $2n + 5\sin\frac{n\pi}{4} - 3a^4$, (2) $\frac{1}{2}(n-1)(n+2)$.	(05)
	b.	If $U(z) = \frac{2z^2+5z+14}{(z-1)^4}$, then find value of u_2 and u_3.	(05)
	c.	Find $L^{-1}\left\{\log\left(1 + \frac{a}{s}\right)\right\}$.	(02)
	d.	If $Z(u_n) = U(z)$, then show that $Z(a^{-n}u_n) = U(az)$.	(02)

OR

Q-6 **Attempt all questions** **(14)**

a. Find the inverse Z-transforms of $\frac{2z\,(2z-1)}{z^3-5z^2+8z-4}$. (05)

b. Given the set of functions $1, x, x^2, x^3$ … . Obtain from these a set of functions which are mutually orthonormal in $(-1, 1)$. (05)

c. If $Z(u_n) = U(z)$, then show that $Z(u_{n-k}) = z^{-k}\, U(z), k > 0$. (02)

d. Evaluate $L[(t+2)^2 e^{6t}]$. (02)